The new method proposed by Suchy [1] belongs to the same class as (7). Stipulating real x, t, k, the ray equations are presented in the form

$$\frac{dx}{dt} = -\mathrm{Re}\,\frac{\partial F/\partial k}{\partial F/\partial \omega},$$

$$\frac{dk}{dt} = \mathrm{Re}\,\frac{\partial F/\partial x}{\partial F/\partial \omega}$$

$$\frac{d\omega}{dt} = -\frac{\partial F/\partial t}{\partial F/\partial \omega} + i\,\mathrm{Im}\left[\left(\frac{\partial F/\partial k}{\partial F/\partial \omega}\right)^* \cdot \frac{\partial F/\partial x}{\partial F/\partial \omega}\right] \tag{8}$$

where the asterisk denotes the complex conjugate. In the models (7), (8), partial derivatives of (1) are used in a formal way. In view of the nonanalytic structure of the ray equations, this needs clarification.

III. An Alternative Model for Real k Rays

Starting with (1) and (2), we define

$$\frac{dk}{dt} = \frac{\partial F/\partial x}{\partial F/\partial \omega} \tag{9}$$

as in (4). Similarly to (5), a condition is imposed

$$\frac{d}{dt}\,\mathrm{Im}\,\frac{\partial F/\partial x}{\partial F/\partial \omega} = \frac{d}{dt}\,\mathrm{Im}\,q = 0$$

$$= \mathrm{Im}\left(\frac{\partial q}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial q}{\partial t} + \frac{\partial q}{\partial k}\cdot\frac{dk}{dt} + \frac{\partial q}{\partial \omega}\,\frac{d\omega}{dt}\right) \tag{10}$$

confining k to the real domain. Solving (2), (10) for dx/dt, $d\omega/dt$ yields

$$\frac{dx}{dt} = -\frac{\partial F/\partial k}{\partial F/\partial \omega} + i\gamma$$

$$\frac{d\omega}{dt} = -\frac{\partial F/\partial t}{\partial F/\partial \omega} + i\gamma\cdot\frac{\partial F/\partial x}{\partial F/\partial \omega}$$

$$\gamma = -\left[\mathrm{Re}\left(\frac{\partial q}{\partial x} + \frac{\partial q}{\partial \omega}\,q\right)\right]^{-1}$$

$$\cdot\,\mathrm{Im}\left(\frac{\partial q}{\partial k}\cdot q - \frac{\partial q}{\partial x}\cdot\frac{\partial F/\partial k}{\partial F/\partial \omega} - \frac{\partial q}{\partial \omega}\,\frac{\partial F/\partial t}{\partial F/\partial \omega} + \frac{\partial q}{\partial t}\right). \tag{11}$$

Again, (9), (11) is a special case of the complex ray tracing method on which the constraint (10) is imposed.

IV. Concluding Remarks

Previous results have been reviewed and a new alternative presented, for ray tracing in absorbing media. If the problem of selecting a single model is to be resolved, numerical and experimental results will be necessary. The question of the analyticity of the dispersion equation seems to play a significant role and should be evaluated.

Appendix

If $f(t) = F(k[t], \omega[t], x[t], t)$ is analytic, then so is df/dt. By inspection of (2) dk/dt, $d\omega/dt$, dx/dt must be analytic too. This is satisfied by (3), (4). In (6), (11) the nonanalytic parts appear in such a way that they are cancelled on substitution into (2).

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Large Signal Instability in Active *RC* Biquadratic Filter Building Blocks

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Abstract—**A comprehensive report on the investigation of large signal instability in second-order high-Q active-RC filters is given. The condition for oscillation and formulas for evaluating the frequencies are derived. Among the several filter building blocks considered, the ones, which are unconditionally stable, are indicated.**

Many of the high-Q active-RC filters using operational amplifiers (OA's) are found to exhibit unstable modes of operation restricting the dynamic range of the filter, when the pole frequency, w_p or the signal level exceed beyond certain value. Often they lock into large signal oscillation, as soon as the supply voltages for the OA's are switched ON. This phenomenon is mainly due to the nonlinearity of the dynamic characteristics of the OA, namely slewing. According to Antoniou [1], the filter attains the unstable mode, while the amplifier gains, which are rising from zero just after activation, reach certain combination. Once it attains the unstable mode, amplitude of the resultant oscillation can rise to a sufficient level to saturate the OA's, preventing further increase in the gain and the filter gets locked into the unstable mode. In this letter, a method for analyzing active-RC filter circuits for large signal instability is presented, in which the filter is initially assumed to be under unstable mode and then the frequency of oscillation and the condition for oscillation are found out. If no such solution is obtained, the filter is presumed to be unconditionally stable.

Stability of the active-RC filter is normally assessed by observing the enhancement in the pole-Q of the filter Q_p due to the finite gain-bandwidth product (GB) of the OA's used. Actual pole-frequency w_p' and actual pole-Q, Q_p', for any active-RC filter can be in general expressed as

$$w_p' \simeq w_p/(1 + a w_p/\mathrm{GB}) \tag{1}$$

$$Q_p' \simeq Q_p/(1 - b\, Q_p w_p/\mathrm{GB}) \tag{2}$$

where a and b are factors which depend on the type of filter configurations. The filter may lock into oscillation, if $w_p \geqslant (\mathrm{GB}/bQ_p)$. With active-compensation schemes [4]–[8], a and b can be reduced considerably, so that w_p' and Q_p' of the filter are very nearly equal to their ideal values. For example, in the double integrator filter using 6 OA's, proposed recently [8], a and b tend towards zero and therefore filter performance is virtually independent of the GB's of the OA's. However all the compensated schemes work satisfactorily only at small signal levels, where the effect of the slew rate of the OA is negligible. At higher signal levels, their performances are mainly decided by the slew rate, which can cause Q_p-enhancement and possibly drive the circuits into unstable modes of operation. Most of the active compensated filter configurations are found to have large signal instability. In this letter, a general procedure is described by which large signal instability of all

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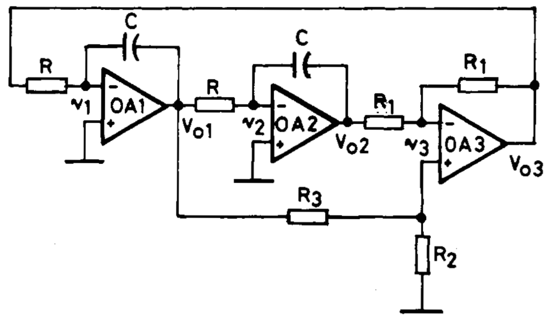

Fig. 1. KHN double integrator filter with input terminal grounded.

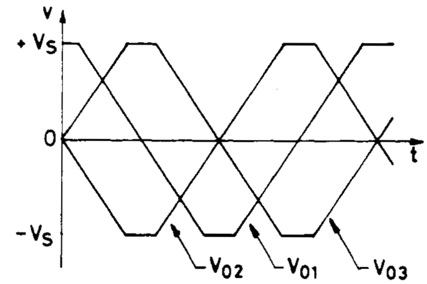

Fig. 2. Large signal oscillatory waveforms at the OA outputs.

TABLE I

Type no. of the filter	Classification of the filters	Type of the filter configurations	Frequency of the large signal oscillation
1	Band pass filter using gyrator circuit	Antoniou [2]	$(\rho/2V_s)/(1+4\rho/w_pV_s)$
2		Kerwin et al [3]	
3		Brown et al [4]	$(\rho/4V_s)/(1+\rho/w_pV_s)$
4	Multifunction biquads	Akerberg et al [5]	
5		Rao et al [7]	$(\rho/4V_s)/(1.25+\rho/w_pV_s)$
6		Ravichandran et al [8]	$(\rho/6V_s)/(1+\rho/1.5w_pV_s)$

active-*RC* filter circuits using OA's can be checked. Simple formulas are derived for computing the frequencies of such large signal oscillation in the case of six popular filter configurations.

For illustration, an uncompensated KHN double integrator filter [3] alone is considered here for brevity. The same method can be extended to any other filter configuration. Referring to the filter circuit shown in Fig. 1, the inputs to OA_1, and OA_2, v_1, and v_2, can be related to their outputs, V_{01}, and V_{02}, by two first-order differential equations as

$$(1/w_p)\frac{dv_1}{dt} + v_1 = (1/w_p)\frac{dV_{01}}{dt} + V_{03} \tag{3}$$

$$(1/w_p)\frac{dv_2}{dt} + v_2 = (1/w_p)\frac{dV_{02}}{dt} + V_{01} \tag{4}$$

where $w_p = 1/RC$. For high-Q_p case, OA_3 functions as an inverting amplifier and therefore phase difference between V_{02} and V_{03} can be assumed to be equal to 180°. Oscillation takes place if each of the integrators represented by OA_1 and OA_2 produces a phase shift of 90°, so that, total phase-shift around the loop remains as 0° (360°). When the filter is locked into large signal oscillation, output of all the OA's can be either at the saturation voltage, $\pm V_s$, or in the transition state from $\pm V_s$ to $\mp V_s$, with the change over taking place at the slew rate ρ. Therefore, waveforms of V_{01}, V_{02} and V_{03} are as shown in Fig. 2 and the half-period of oscillation ($T/2$) becomes equal to

$$T/2 = 2V_s/\rho + \Delta t \tag{5}$$

where Δt is the duration in which the output of the OA remains at $\pm V_s$. Δt and the condition for oscillation are obtained from the steady state solutions of (3) and (4), satisfying the input–output relationships of the voltages of the OA's for each segment of the waveform, shown in Fig.2

$$\Delta t = 2/w_p \tag{6}$$

Condition for oscillation:

$$w_p > \rho/V_s. \tag{7}$$

From (5) and (6), the frequency of oscillation f_0 is obtained as

$$f_0 = (\rho/4V_s)/(1 + \rho/w_pV_s). \tag{8}$$

On similar lines, analysis can be carried out for eight other filter configurations, which include two bandpass filters using Antoniou's gyrator circuits [2] and six multifunction biquads [4]-[8]. Of these, two circuits, namely a bandpass filter using one of the gyrators [2] and a compensated double integrator filter [6], are found to be unconditionally stable. All the remaining configurations have unstable modes of operation for $w_p > \rho/V_s$. Formulas for computing the frequencies of large signal oscillation in these circuits are shown in Table I.

TABLE II

Type no. of the filter	Supply voltage of the OA	Frequency of the large signal oscillation f_0(KHz)					
		w_p= 9.3 KHz		w_p= 16.0KHz		w_p =27.7KHz	
		Theoretical	Experimental	Theoretical	Experimental	Theoretical	Experimental
1		5.7	7.7	8.5	9.0	15.8	14.0
2			7.1		7.9		8.8
3	± 10V	6.7	8.2	8.3	8.3	9.7	9.8
4			8.2		8.4		9.9
5		5.9	8.4	7.2	8.1	8.1	8.2
6		5.3	5.0	6.3	5.7	6.9	6.3
1		5.1	5.8	7.2	8.5	11.8	13.2
2			5.3		6.0		6.6
3	± 15V	5.2	5.6	6.1	6.3	6.8	7.7
4			5.6		6.3		7.8
5		4.5	5.3	5.1	5.6	5.6	5.9
6		3.9	3.9	4.4	4.3	4.8	4.7

All the filter circuits were built using dual OA's(ML 747) with R = 1.794 kΩ and C corresponding to the w_p chosen and tested for large signal oscillations at Q_p = 105.3 with different w_p's (9.3 kHz, 16.0 kHz, and 27.7 kHz) and with different supply voltages (±10 V and ±15 V). The OA's used, were found to have ρ = 0.4 V/μs and V_s = ±8 V with a supply voltage of ±10 V and ρ = 0.42 V/μs and V_s = ±13 V with a supply voltage of ±15 V. All the resistors and capacitors in the circuit, were of tolerances ±0.5 percent and ±1 percent, respectively. Frequencies of large signal oscillation in those filters with unstable modes of operation, are shown in Table II. The experimental results are in close agreement with the theoretical predictions, except for the slight discrepancy at w_p = 9.3 kHz, where the condition for oscillation (7) is just satisfied. Investigation here, has been restricted to the large signal instability, in which full voltage swing (± V_s) is obtained at the output of the OA. Signal-limiting diodes [1], [5]-[8], therefore become necessary to prevent the unstable modes of operation in the circuits listed in the tables.

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Comments on "Index Profile Measurement of Fibers and Their Evaluation"

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From (16) of the above titled paper[1] follows, for a Lambert law source, $S(\theta) = S \cos\theta$

$$S_1 = S_2 \frac{\cos^2\theta_2}{\cos^2\theta_1} \frac{n_1^2}{n_2^2}$$

which, if it were correct, would imply that the radiance of a thermal source could be enhanced indefinitely.

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